2967. Unification day

Byteland has *n* cities, but no one road. The king of the country, Waldemar de Bear, decided to remedy this situation, he wants to connect some cities with roads so that along these roads it will be possible to reach any city from any other city. When construction will be completed, the king plans to celebrate the Unification Day.

Unfortunately, the treasury in Byteland is almost empty, so the king needs to save money and minimize the total length of constructed roads.

Input. The first line contains the number n ($1 \le n \le 5000$) of cities in Byteland. Each of the next n lines contains two integers x_i, y_i, the coordinates of i-th city ($-10000 \le x_i, y_i \le 10000$). No two cities are located in one point.

Output. Print the least total roads length. Print the answer with the accuracy no less than 10-3.

Sample input	Sample output
6 1 1 7 1 2 2 6 2 1 3 7 3	9.6568542495

SOLUTION

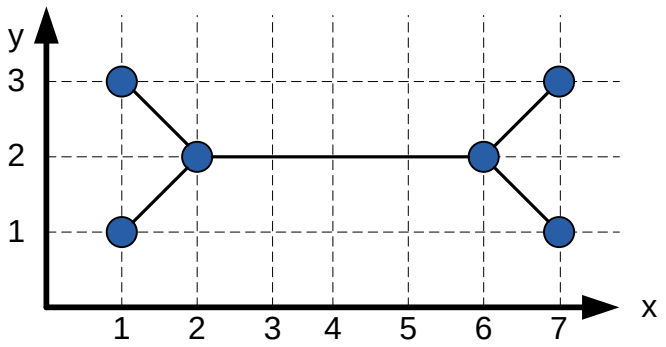
graphs – minimum spanning tree – Prim

Algorithm analysis

To solve the problem, you need to find the minimum spanning tree (MST). Implement Prim algorithm.

Example

Construct the minimum spanning tree for the graph given in the sample.



The weight of the minimum spanning tree is $4 + 4\sqrt{2} \approx 9.65$.

Algorithm realization

Declare global arrays. Store the coordinates of the cities in (x[*i*], y[*i*]). used[*i*] = 1, if vertex *i* is included in MST.

```
#define MAX 5001
#define INF 0x3F3F3F3F
int x[MAX], y[MAX];
int used[MAX], dist[MAX];
```

The function ***dist2*** computes the squared distance between cities *i* and *j*.

```
int dist2(int i, int j)
{
  return (x[j] - x[i])*(x[j] - x[i]) + (y[j] - y[i])*(y[j] - y[i]);
}
```

Function ***Prim*** implements the Prim's algorithm and returns the value of MST.

```
double Prim(void)
{
```

Initialize the arrays.

```
  memset(dist, 0x3F, sizeof(dist));
  memset(used, 0, sizeof(used));
```

Start to construct the MST from the vertex 0. Initialize dist[0] = 0, used[0] = 1. In the variable *res* compute the size of MST.

```
  double res = 0;
  int i, j, cur = 0;
  dist[cur] = 0;
  used[cur] = 1;
```

Make *n* – 1 iterations. At each iteration, add one vertex to MST (so *n* – 1 vertices should be added to MST).

```
  for (i = 1; i < n; i++)
  {
```

The current vertex is *cur*. Iterate over the edges (*cur*, *j*) and recalculate the value of dist[*j*]. Thus dist[*j*] stores the current shortest distance from vertex *j* to the current MST.

```
    for (j = 0; j < n; j++)
      if (!used[j] && (dist2(cur, j) < dist[j]))
        dist[j] = dist2(cur, j);
```

Find the shortest edge that will be included in MST. To do this, look for the smallest dist[*j*] such that the vertex *j* does not yet belong to MST (used[*j*] = 0). The number of the vertex with the smallest dist[*j*] is stored into *cur* (it becomes the current one).

```
    int min = INF;
    for (j = 0; j < n; j++)
      if (!used[j] && (dist[j] < min))
      {
        min = dist[j];
        cur = j;
      }
```

Vertex *cur* is included to MST. The edge of length ***sqrt***(dist[*cur*]) is added to MST.

```
    used[cur] = 1;
    res += sqrt(dist[cur]);
  }
  return res;
}
```

The main part of the program. Read the input data.

```
scanf("%d", &n);
for (i = 0; i < n; i++)
  scanf("%d %d", &x[i], &y[i]);
```

Run the Prim's algorithm and print the answer.

```
res = Prim();
printf("%lf\n", res);
```

Algorithm realization – min_e / end_e

Declare the arrays. Store the city coordinates in (x[*i*], y[*i*]).

```
#define MAX 5001
int x[MAX], y[MAX];
int used[MAX], min_e[MAX], end_e[MAX];
```

The function ***dist2*** computes the squared distance between cities *i* and *j*.

```
int dist2(int i, int j)
{
  return (x[j] - x[i])*(x[j] - x[i]) + (y[j] - y[i])*(y[j] - y[i]);
}
```

The main part of the program. Read the coordinates of the cities.

```
scanf("%d",&n);
for(i = 0; i < n; i++)
  scanf("%d %d",&x[i], &y[i]);
```

Initialize the arrays.

```
memset(min_e,0x3F,sizeof(min_e));
memset(end_e,-1,sizeof(end_e));
memset(used,0,sizeof(used));
```

The size of MST will be calculated in the *dist* variable.

```
dist = min_e[1] = 0;
for (i = 0; i < n; i++)
{
```

Look for the vertex *v* with minimum value of min_e[*v*] among the vertices that are not yet included in MST (for which used[*v*] = 0).

```
  v = -1;
  for (j = 0; j < n; j++)
    if (!used[j] && (v == -1 || min_e[j] < min_e[v])) v = j;
```

Include the vertex *v* into MST. Add an edge (*v*, end_e[*v*]) to MST.

```
  used[v] = 1;
  if (end_e[v] != -1) dist += sqrt((double)dist2(v,end_e[v]));
```

Recompute the labels for the edges outgoing from *v*.

```
  for (to = 0; to < n; to++)
  {
    int dV_TO = dist2(v,to);
    if (!used[to] && (dV_TO < min_e[to]))
    {
      min_e[to] = dV_TO;
      end_e[to] = v;
    }
  }
}
```

Print the value of MST.

```
printf("%.6lf\n",dist);
```

Java realization

```
import java.util.*;

public class Main
{
  static int x[], y[];
  static int used[], min_e[], end_e[];

  static int dist2(int i, int j)
  {
    return (x[j] - x[i])*(x[j] - x[i]) + (y[j] - y[i])*(y[j] - y[i]);
  }

  public static void main(String[] args)
  {
    Scanner con = new Scanner(System.in);
    int n = con.nextInt();
    x = new int[n];
    y = new int[n];
    for(int i = 0; i < n; i++)
    {
```

```
      x[i] = con.nextInt();
      y[i] = con.nextInt();
    }

    min_e = new int[n];
    Arrays.fill(min_e, Integer.MAX_VALUE);
    end_e = new int[n];
    Arrays.fill(end_e, -1);

    used = new int[n];

    double dist = min_e[1] = 0;
    for (int i = 0; i < n; i++)
    {
      int v = -1;
      for (int j = 0; j < n; j++)
        if (used[j] == 0 && (v == -1 || min_e[j] < min_e[v])) v = j;

      used[v] = 1;
      if (end_e[v] != -1)
        dist += Math.sqrt((double)dist2(v,end_e[v]));

      for (int to = 0; to < n; to++)
      {
        int dV_TO = dist2(v,to);
        if (used[to] == 0 && (dV_TO < min_e[to]))
        {
          min_e[to] = dV_TO;
          end_e[to] = v;
        }
      }
    }

    System.out.println(dist);
    con.close();
  }
}
```